Comment on “Self-organized criticality in living systems” by C. Adami

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Abstract

Following extensive numerical experiments, it has been suggested that the evolution of competing computer programs in artificial life simulations shows signs of being a self-organized critical process. The primary evidence for this claim comes from the distribution of the lifetimes of species in the simulations, which appears to follow a power law. We argue that, for a number of reasons, it is unlikely that the system is in fact at a critical point and suggest an alternative explanation for the power-law lifetime distribution.

In two recent papers, Adami [1,2] has presented results from extensive simulations using the Tierra artificial life system, which appear to show evidence of self-organized critical behaviour in an evolving system. Should it turn out to be correct, this observation would be of considerable interest in evolutionary theory, where the idea of self-organization has been a topic of debate for some years now. In this comment, however, we would like to question this interpretation of the Tierra data. For a number of reasons, we believe it unlikely that one would be able to see critical phenomena in a system such as Tierra, and furthermore there are certain features of the data which strongly suggest that the system is not in a critical state. We therefore propose an alternative interpretation which, we believe, explains the observed behaviour of the system without assuming criticality.

The fundamental idea behind self-organized critical models of evolving systems is that of coevolutionary avalanches [3]. It is known that species interaction through such mechanisms as competition, predation, parasitism and symbiosis can cause one species to evolve as a result of the evolution of another. It is possible therefore for the chance mutation of a species in a large ecosystem

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to spark off a chain, or “avalanche”, of evolution affecting a large number of other species and potentially the entire ecosystem. Models of this process have been proposed, notably those of Kauffman and Johnsen [4] and of Bak and Sneppen [5], which predict that the resulting reorganizations of the ecosystem will drive it towards a critical state in which the distribution of avalanche sizes follows a power law.

Coevolutionary avalanches have not been directly observed in Tierra. The primary evidence for criticality comes instead from the distribution of “epoch” lengths in the evolution of the ecosystem. If we record the fitness (which can be defined roughly as the reproduction rate) of the fittest species in Tierra as a function of time, the measured values increase not smoothly, but in jumps, which arise as the result of evolutionary innovations in the population. Adami [1] has dubbed these jumps “phase transitions”, by analogy with the first order phase transitions of statistical physics. Immediately before an innovation the system can be considered to be in a metastable state, waiting for the fluctuation which will take it over some barrier to a state of lower free energy. The transition itself proceeds precisely by a nucleation process, just as do many more familiar transitions. One individual in the population discovers the trick which allows it to reproduce more efficiently and, like the freezing of super-cooled water, the offspring of that individual spread through the ecosystem causing a sudden jump in the mean fitness of the population. (Notice though that in Tierra the jump is merely to another metastable state which will ultimately itself give way to a still more favourable one.) If one constructs a histogram of the time intervals τ, also called epochs, between each jump and the next, the resulting curve follows a power-law distribution whose exponent is measured to be $\alpha = 1.10 \pm 0.05$ [1]. Power-law distributions are often taken to be indicative of self-organized critical behaviour, and indeed the evolution model of Bak and Sneppen [5] predicts a power law of precisely this type in the lengths of the epochs separating consecutive avalanches. Do the data therefore indicate that the Tierra system evolves to a self-organized critical state? We have a number of objections to this interpretation.

(i) The jumps, or phase transitions, seen in Tierra are not equivalent to coevolutionary avalanches. Whilst avalanches arise through the interaction of many species, the phase transitions here are essentially the product of just one species, whose fitness exceeds that of all others, allowing it rapidly to dominate the system.

(ii) The phase transitions are strongly first order, as is clear from the presence of fast nucleation processes. Critical phenomena are not observed in the vicinity of strongly first order transitions.

(iii) In our own investigations of the Tierra system, we observe that interspecies interactions are relatively rare. On average each species interacts with

less than one other. However, each species must interact with at least two others in order to establish a percolating interaction network. Such a network is necessary if system-spanning coevolutionary avalanches are to take place, and therefore we conclude that avalanches of this kind are not possible in Tierra. In real ecosystems it is estimated that each species interacts with between three and four others on the average [6], a figure more consistent with the coevolutionary avalanche picture. (We can neglect the trivial interaction that arises because all species in Tierra compete for space and CPU time—this interaction produces correlations between the birth and death rates of different species, but does not give rise to coevolution.)

(iv) The Tierran ecology at any one moment tends to be dominated by the fittest species, or the fittest handful of species. Even ignoring points (i), (ii) and (iii), this would limit the size of possible avalanches to just a few species. In effect, the finite-size effects on the system would truncate any critical behaviour so that we would not expect it to be visible in the simulations.

So, if the Tierra system is not in a critical state, how are we to account for the appearance of the power law? One possible explanation is that Tierran evolution is an "extremal random process". Suppose that at some point during a simulation the process of finding a genotype which is fitter than the current fittest one in the system takes a time t_1. How long then will it take to find another one which is fitter still? If the sampling of fitness by mutation and selection is a random process, we can assume that it will on average take another time t_1 to find a new genotype with the same or greater fitness, or an aggregate time of $t_2 = 2t_1$. This result will be true regardless of the distribution over fitnesses which we sample—for instance, it makes no difference if we sample genotypes of lower fitness more often than those of higher fitness, the result will still be the same. Iterating the argument, the time taken to find the next fitter genotype will be $t_3 = 2t_2 = 4t_1$ and so forth. The time t_n at which the n^{th} such innovation occurs is then given by

$$t_n = t_1 2^n, \tag{1}$$

and the duration of the corresponding epoch is

$$\tau = t_{n+1} - t_n = t_1(2^{n+1} - 2^n) = t_1 2^n. \tag{2}$$

Thus the number of epoch lengths $\mathrm{d}n$ falling, on average, in an interval $\mathrm{d}\tau$ is

$$\mathrm{d}n = p(\tau)\mathrm{d}\tau = \frac{2\mathrm{d}\tau}{n\tau}. \tag{3}$$

Equation (2) tells us that the n in the denominator here goes like $\log \tau$ and hence, apart from logarithmic corrections, the distribution of epochs $p(\tau)$ goes

like τ^{-1}. As we mentioned, the actual measured behaviour is $\tau^{-\alpha}$ with $\alpha = 1.10 \pm 0.05$, so this result is in reasonable agreement with the simulations.

An alternative explanation is that the Tierra system contains "fitness barriers" across which species have to pass by mutation in order to reach genotypes of higher fitness. Models such as that of Bak and Sneppen [5] assume that barrier crossing is a thermal excitation process with a typical crossing time τ (equivalent to the epoch lengths above) which is exponentially related to the barrier height B:

$$\tau \propto \mathrm{e}^{-B/T}, \tag{4}$$

where T is some temperature-like parameter whose exact value we do not know. If we assume that the barriers are randomly distributed according to some probability distribution $p_{\mathrm{barrier}}(B)$, then the distribution of τ is given by

$$p(\tau) = p_{\mathrm{barrier}}(B)\frac{\mathrm{d}B}{\mathrm{d}\tau} \sim \frac{p_{\mathrm{barrier}}(\log\tau)}{\tau}. \tag{5}$$

So again, apart from logarithmic corrections, we find a distribution of epoch lengths which goes like τ^{-1}, in agreement with the data.

Either of these two scenarios is capable of explaining the behaviour observed in Tierra without assuming self-organized criticality. It would be more satisfying however if we were able to say with some degree of certainty which one we believe to be correct. In the first scenario we can use Equation (1) to predict the distribution of the jumps as a function of the elapsed time during the simulation. Following precisely the same argument as we employed to write Equation (3) we can then show that, if the first scenario is correct, the timing of the jumps should, like the epoch lengths, follow a power law of t^{-1}. Conversely, the distribution of jumps as a function of elapsed time in the second scenario is entirely uniform. Appropriate analysis of the data from the simulations performed by Adami should therefore allow us to distinguish between the two cases.

To conclude, we believe for a number of reasons that it is unlikely that one can observe critical behaviour in a system such as Tierra. The observed power-law distribution of epoch times we explain instead as a result of the random sampling of genotypes by the mutation and selection processes, or possibly as the result of the way in which the system traverses fitness barriers. There is, nonetheless, much of interest in the simulations carried out by Adami. In particular, we believe that the sudden jumps seen in the fitness constitute one of the clearest examples yet of the phenomenon known as "punctuated equilibrium" [7]. It is well known amongst palaeontologists that, rather than evolving continually in small phenotypic jumps, fossil species tend to remain

unchanged for many millions of years, before suddenly undergoing evolution to a new form. An explanation of this effect was given by George Simpson in 1944 [8] who pointed out that behaviour of exactly this type is to be expected of species whose evolution to a fitter form requires them to cross some fitness barrier, either energetic or entropic in nature. Tierran organisms fall into precisely this category, so it is satisfying to see punctuated behaviour so clearly in evidence.

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